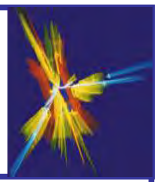
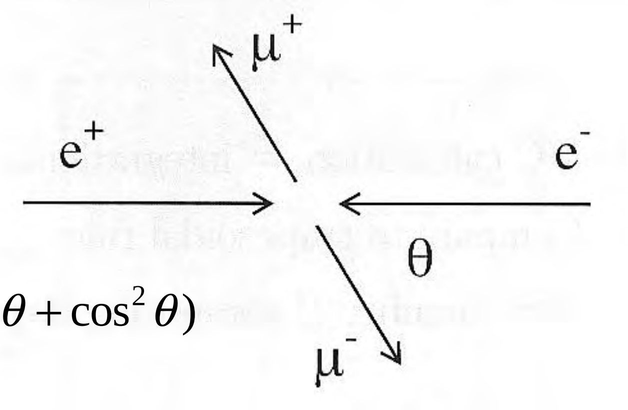


MC methods often used to ”mimic” data in e.g. particle physics, a two-step process: simulation of interaction, ”**event generator**”, & simulation of detector response, ”**detector simulation**”.

<u>MC event generator:</u>
simple example:
$e^+e^- \to \mu^+\mu^-$



generate θ and ϕ:

$$f(\cos\theta; A_{FB}) \propto (1 + \tfrac{8}{3} A_{FB} \cos\theta + \cos^2\theta)$$

$$g(\phi) = 1/2\pi$$

In reality interactions implemented into program packages (that take into account hadronisation, intial & final state radiation…)

<u>MC detector simulation (e.g. GEANT4):</u>
takes as input the particle momenta from event generator

simulate detector response: multiple Coulumb scattering (generate scattering angle), particle decays (generate lifetime), ionization energy loss (generate energy loss), electromagnetic/ hadronic showers, generation of the detector signal, ...

output = simulated raw data $\to$ input to the reconstruction.

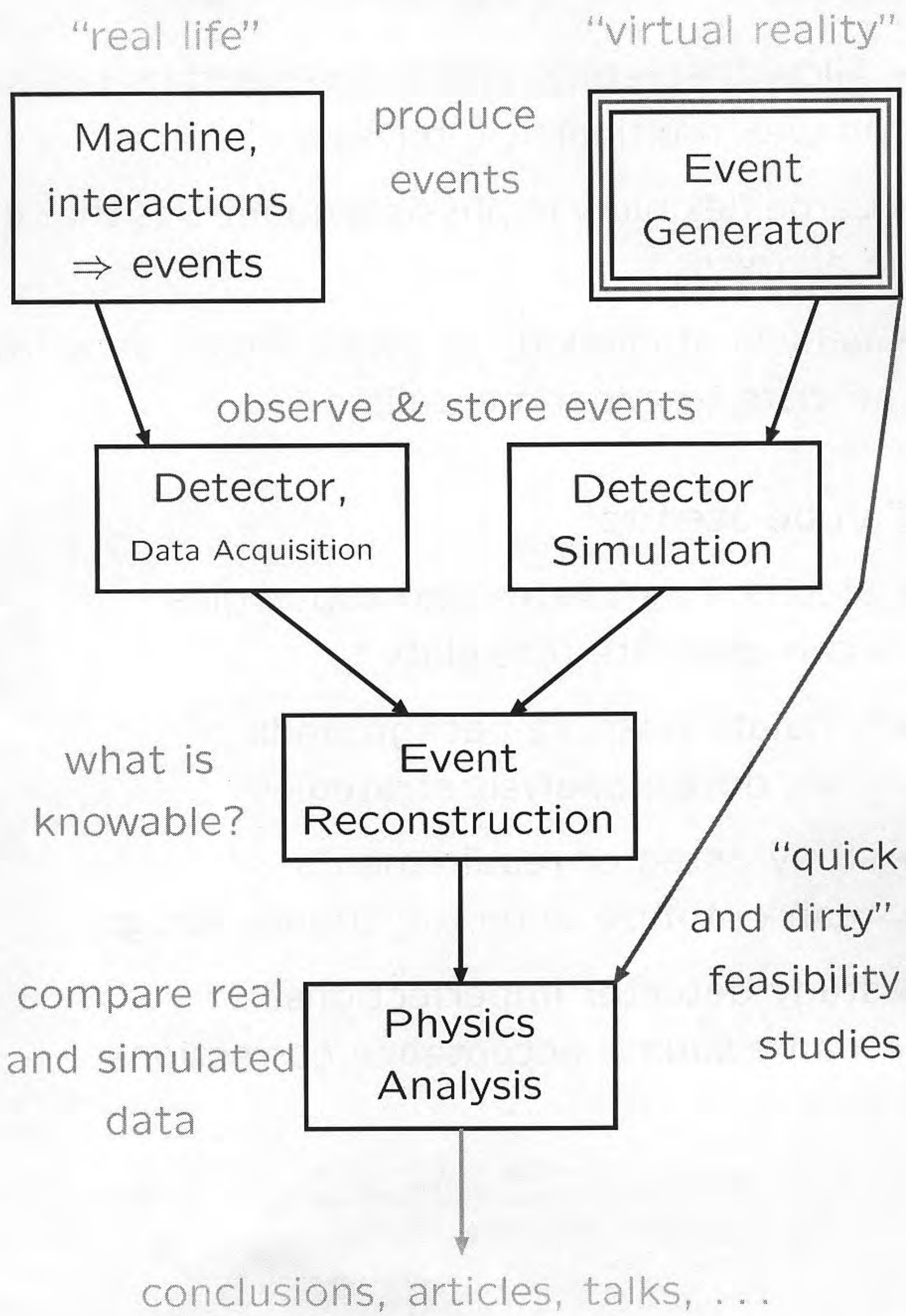
<u>Use:</u>
Predict what you should see at ”detector level” given a certain hypothesis at ”generator level”. Compare this with real data.

Estimate various ”efficiencies” = # events found / # events generated

Optimisation of experimental design & study of physics reach.

Event Generator Position

(T. Sjöstrand)



Why Generators?

(T. Sjöstrand)

- Allow theoretical and experimental studies of *complex* multiparticle physics
- Large flexibility in physical quantities that can be addressed
- Vehicle of ideology to disseminate ideas from theorists to experimentalists

Can be used to

- predict event rates and topologies
  $\Rightarrow$ can estimate feasibility
- simulate possible backgrounds
  $\Rightarrow$ can devise analysis strategies
- study detector requirements
  $\Rightarrow$ can optimize detector/trigger design
- study detector imperfections
  $\Rightarrow$ can evaluate acceptance corrections

God does not throw dice . . .
. . . but Mother Nature does!

Generator Landscape

(T. Sjöstrand)

	General-Purpose	Specialized
Hard Processes	HERWIG, PYTHIA, ISAJET, SHERPA	a lot
Resonance Decays		HDECAY, . . .
Parton Showers		Ariadne/LDC, NLLjet
Underlying Event		DPMJET
Hadronization		none (?)
Ordinary Decays		TAUOLA, EvtGen

specialized often best at given task, but need General-Purpose core

HERWIG

http://hepwww.rl.ac.uk/theory/seymour/herwig/

PYTHIA

http://www.thep.lu.se/∼torbjorn/Pythia.html

ISAJET

http://www.phy.bnl.gov/∼isajet/

SHERPA

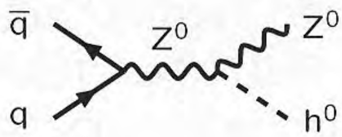
http://www.physik.tu-dresden.de/∼krauss/hep/

Event Physics Overview

(T. Sjöstrand)

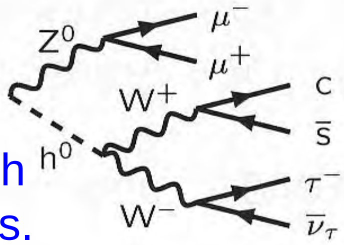
Structure of the basic generation process:

1) Hard subprocess:
$d\hat{\sigma}/d\hat{t}$, Breit-Wigners.

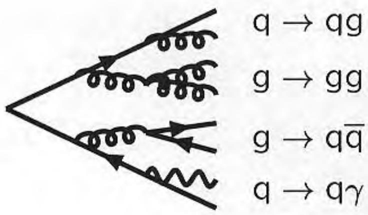


2) Resonance decays:
includes correlations.

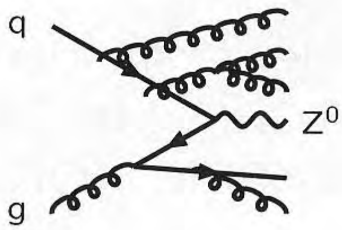
e.g. angular. Deals essentially with electroweak or beyond SM decays.



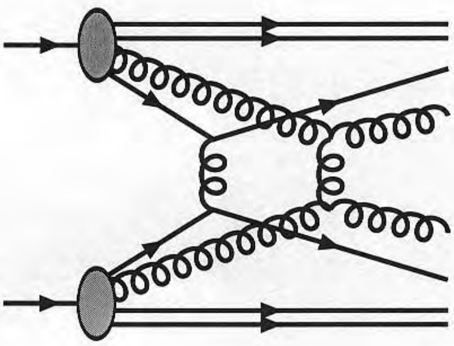
3) Final-state
parton showers:
(or matrix elements).

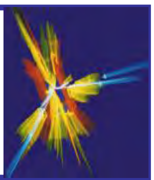


4) Initial-state
parton showers:
(or matrix elements).



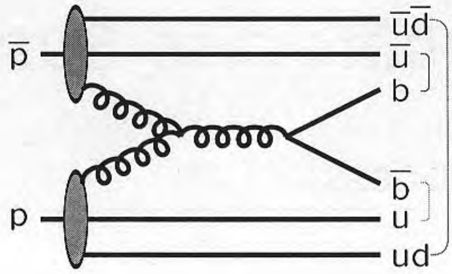
5) Multiple
parton–parton
interactions.



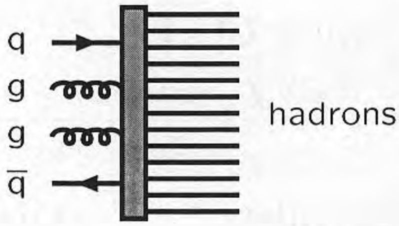


(T. Sjöstrand)

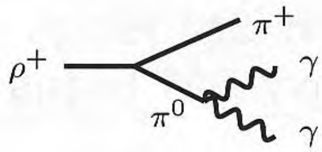
6) Beam remnants: colour-connected to rest of event



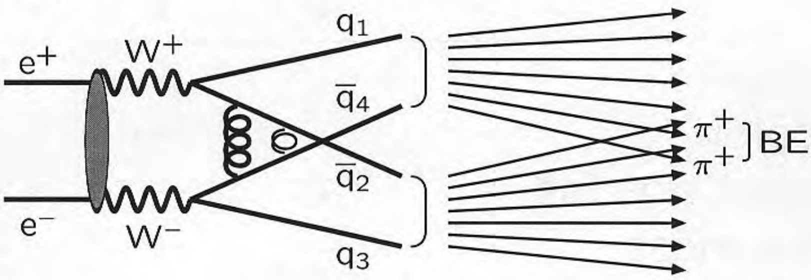
7) Hadronization (PYTHIA: string; HERWIG: cluster; ISAJET: independent).



8) Normal decays: hadronic, τ, charm, . . .

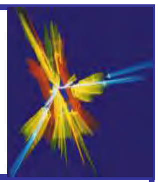


9) QCD interconnection effects:



a) colour rearrangement ($\Rightarrow$ rapidity gaps?);
b) Bose-Einstein.

10) The forgotten/unexpected: a chain is never stronger than its weakest link!



The Smaller Picture: Subprocess Survey

(T. Sjöstrand)

Kind	Process	PYT	HER	ISA
QCD & related	Soft QCD	⋆	⋆	⋆
	Hard QCD	⋆	⋆	⋆
	Heavy flavour	⋆	⋆	⋆
Electroweak SM	Single $\gamma^*/Z^0/W^\pm$	⋆	⋆	⋆
	$(\gamma/\gamma^*/Z^0/W^\pm/f/g)^2$	⋆	⋆	⋆
	Light SM Higgs	⋆	⋆	⋆
	Heavy SM Higgs	⋆	⋆	⋆
SUSY BSM	$h^0/H^0/A^0/H^\pm$	⋆	⋆	⋆
	SUSY	⋆	⋆	⋆
	R̸ SUSY	⋆	⋆	—
Other BSM	Technicolor	⋆	—	(⋆)
	New gauge bosons	⋆	—	—
	Compositeness	⋆	—	—
	Leptoquarks	⋆	—	—
	$H^{\pm\pm}$ (from LR-sym.)	⋆	—	—
	Extra dimensions	(⋆)	(⋆)	(⋆)

e.g. PYTHIA includes following Standard Model processes:

- Hard QCD processes e.g. $qg \to q\gamma$.
- Soft QCD processes: diffractive and elastic scattering + minimum bias events.
- Heavy flavour production: both open & hidden e.g. $gg \to t\bar{t}$ & $gg \to J/\psi\, g$.
- Prompt photon production, e.g. $qg \to g\gamma$
- Photon induced processes, e.g. $\gamma g \to q\bar{q}$
- Deep inelastic scattering, e.g. $ql \to ql$
- W/Z production: such as $e^+e^- \to \gamma^*/Z^0$ or $qq \to W^+W^-$

Subprocess lists *look* impressive, and have involved a lot of hard work, but: (T. Sjöstrand)

⋆ Processes usually only in lowest nontrivial order
⇒ need programs that include HO loop corrections to cross sections, alternatively do $(p_\perp, y)$-dependent rescaling by hand?

⋆ No multijet topologies
⇒ have to trust shower to get it right, alternatively match to HO (non-loop) ME generators

⋆ Spin correlations often absent or incomplete
e.g. top produced unpolarized, while $t \to bW^+ \to b\ell^+\nu_\ell$ decay correct
⇒ have to use external programs when important

⋆ New physics scenarios appear at rapid pace
⇒ need to have a bigger class of "one-issue experts" contributing code

⇒ The Les Houches Accord

Specialized Generator ⟹ Hard Process

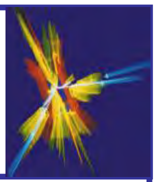
↓

Les Houches Interface

↓

HERWIG or PYTHIA
- (Resonance Decays)
- Parton Showers
- Underlying Event
- Hadronization
- Ordinary Decays

Some Specialized Generators:
- AcerMC: $t\bar{t}b\bar{b}$, ...
- ALPGEN: $W/Z + \leq 6j$, $nW + mZ + kH + \leq 3j$, ...
- AMEGIC++: generic LO
- CompHEP: generic LO
- GRACE+Bases/Spring: generic LO+ some NLO loops
- GR@PPA: $b\bar{b}b\bar{b}$
- MadCUP: $W/Z + \leq 3j$, $t\bar{t}b\bar{b}$
- MadGraph+HELAS: generic LO
- MCFM: NLO $W/Z + \leq 2j$, WZ, WH, $H + \leq 1j$
- O'Mega+WHIZARD: generic LO
- VECBOS: $W/Z + \leq 4j$



Particle data group particle codes (T. Sjöstrand)

A. Fundamental objects

1	d	11	e^-	21	g				
2	u	12	ν_e	22	γ	32	Z'^0		
3	s	13	μ^-	23	Z^0	33	Z''^0		
4	c	14	ν_μ	24	W^+	34	W'^+		
5	b	15	τ^-	25	h^0	35	H^0	37	H^+
6	t	16	ν_τ			36	A^0	39	Graviton

add − sign for antiparticle, where appropriate

+ diquarks, SUSY, technicolor, . . .

B. Mesons

$100\,|q_1| + 10\,|q_2| + (2s+1)$ with $|q_1| \geq |q_2|$

particle if heaviest quark u, $\bar{\text{s}}$, c, $\bar{\text{b}}$; else antiparticle

111	π^0	311	K^0	130	K^0_L	221	η^0	411	D^+	431	D^+_s
211	π^+	321	K^+	310	K^0_S	331	η'^0	421	D^0	443	J/ψ

C. Baryons

$1000\,q_1 + 100\,q_2 + 10\,q_3 + (2s+1)$

with $q_1 \geq q_2 \geq q_3$, or Λ-like $q_1 \geq q_3 \geq q_2$

2112	n	3122	Λ^0	2224	Δ^{++}	3214	Σ^{*0}
2212	p	3212	Σ^0	1114	Δ^-	3334	Ω^-

HEPEVT event record (being replaced by HEPMC in C++)

```
      PARAMETER (NMXHEP=4000)
      COMMON/HEPEVT/NEVHEP,NHEP,ISTHEP(NMXHEP),IDHEP(NMXHEP),
     &JMOHEP(2,NMXHEP),JDAHEP(2,NMXHEP),PHEP(5,NMXHEP),
     &VHEP(4,NMXHEP)
      DOUBLE PRECISION PHEP, VHEP
```

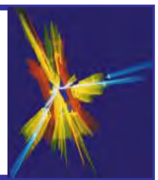
NMXHEP = maximum number of entries
NEVHEP = event number

NHEP = number of entries in current event

ISTHEP = status code of entry (0 = null entry, 1 = existing entry,
2 = fragmented/decayed entry, 3 = documentation entry)
IDHEP = PDG particle identity (+ some internal, e.g. 92 = string)

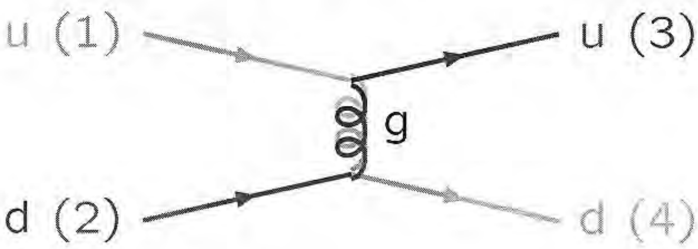
JMOHEP = mother position(s)
JDAHEP = first and last daughter position

PHEP = momentum (p_x, p_y, p_z, E, m) in GeV
VHEP = production vertex (x, y, z, t) in mm



(T. Sjöstrand)

Cross sections and kinematics

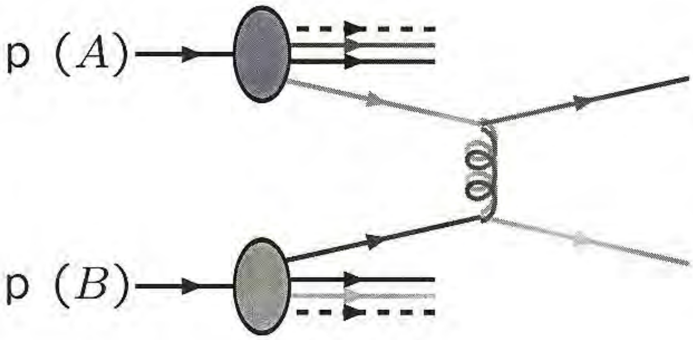


$\hat{s} = (p_1 + p_2)^2$

$\hat{t} = (p_1 - p_3)^2$

$\hat{u} = (p_1 - p_4)^2$

$$\mathrm{qq'} \to \mathrm{qq'} \quad : \quad \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$



$s = (p_A + p_B)^2$

$x_1 \approx E_1/E_A$

$x_2 \approx E_2/E_B$

$\hat{s} = x_1 x_2 s$

$$\sigma = \sum_{i,j} \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \, f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

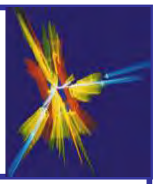
$f_i(x, Q^2)$: parton distribution functions at characteristic scale $Q^2 \approx \mathrm{p}_\perp^2 = \hat{t}\hat{u}/\hat{s}$

luminosity $\quad \mathcal{L} \propto \frac{N_1 N_2 f}{A}$

counting rate $\quad \frac{\mathrm{d}N_{\mathrm{event}}}{\mathrm{d}t} = \sigma \, \mathcal{L}$

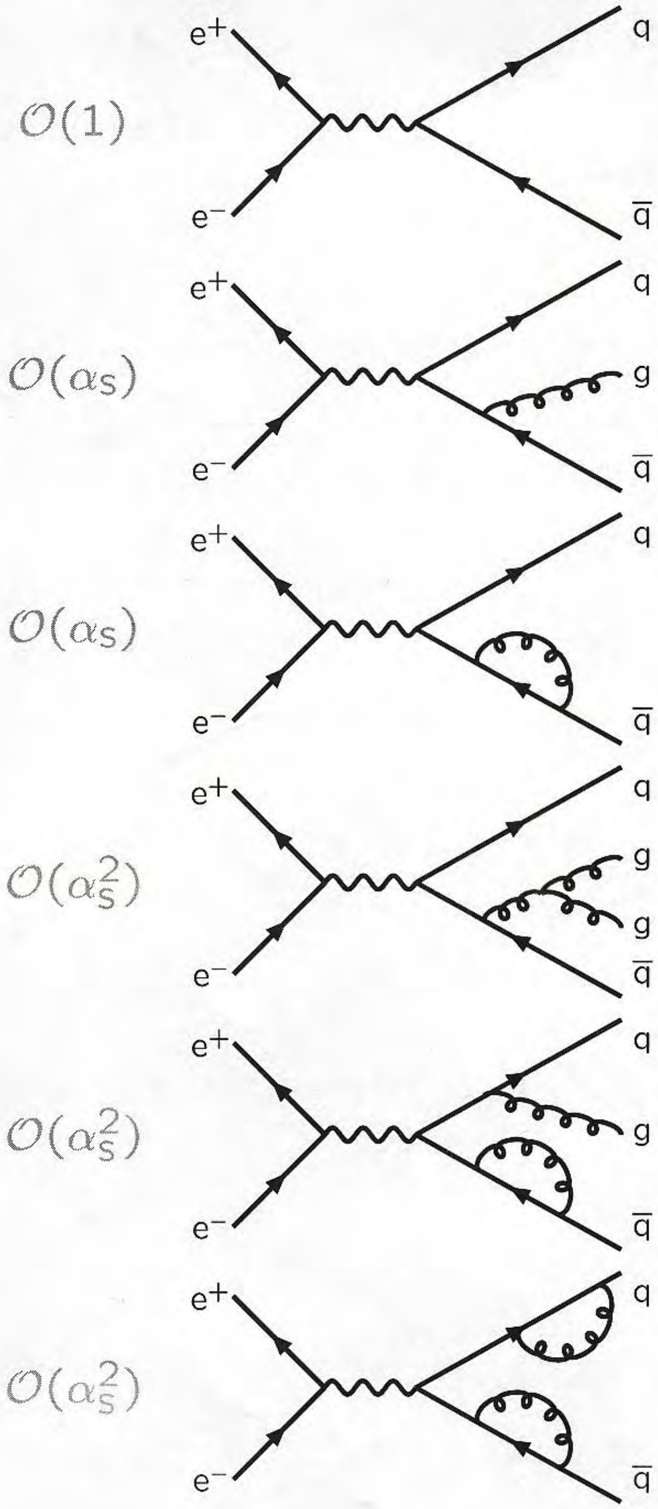
total rate $\quad N_{\mathrm{event}} = \sigma \int \mathcal{L}(t) \, \mathrm{d}t$

Limitations of matrix element



(T. Sjöstrand)

Higher Order Matrix Elements



Matrix Elements exact to given order. . . but blind to higher orders

$\mathcal{O}(\alpha_s L^2)$

$L \simeq -\ln y$

$y \simeq \min \frac{m_{ij}^2}{E_{\mathrm{cm}}^2}$

$\mathcal{O}(\alpha_s^2 L^4)$

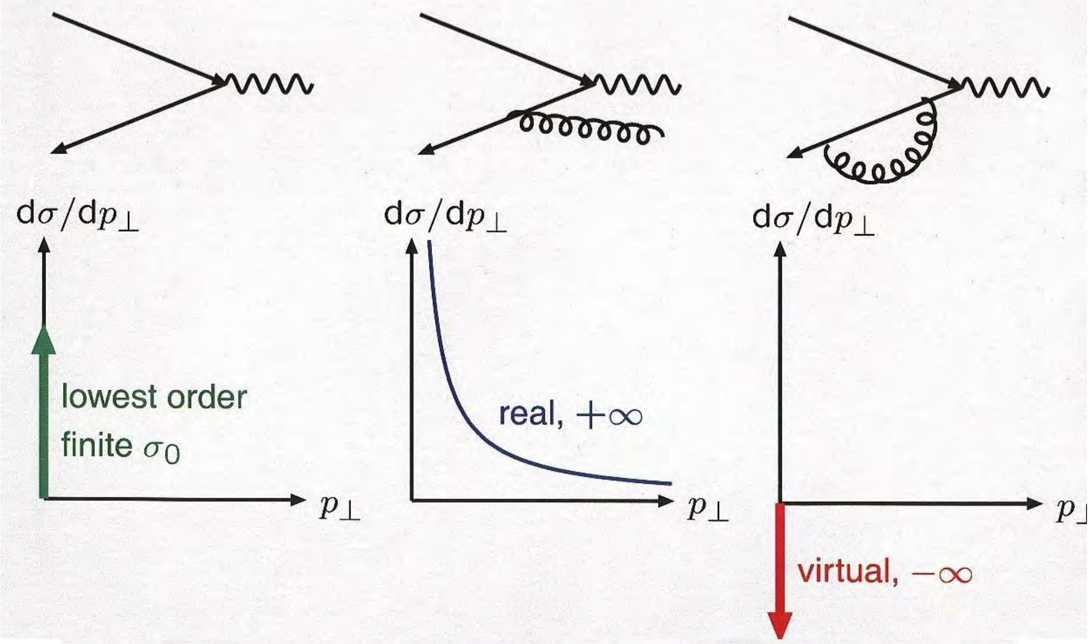
collinear and soft emission divergences $\Rightarrow$ large higher orders

Next-to-leading order (NLO) calculations

I. Lowest order, $\mathcal{O}(\alpha_{\rm em})$: $q\bar{q} \to Z^0$

II. First-order real, $\mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$: $q\bar{q} \to Z^0 g$ etc.

III. First-order virtual, $\mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$: $q\bar{q} \to Z^0$ with loops



$$\sigma_{\rm NLO} = \int_n {\rm d}\sigma_{\rm LO} + \int_{n+1} {\rm d}\sigma_{\rm Real} + \int_n {\rm d}\sigma_{\rm Virt}$$

Simple one-dimensional example: $x \sim p_\perp/p_{\perp\rm max}$, $0 \leq x \leq 1$

Divergences regularized by $d = 4 - 2\epsilon$ dimensions, $\epsilon < 0$

$$\sigma_{\rm R+V} = \int_0^1 \frac{{\rm d}x}{x^{1+\epsilon}} M(x) + \frac{1}{\epsilon} M_0$$

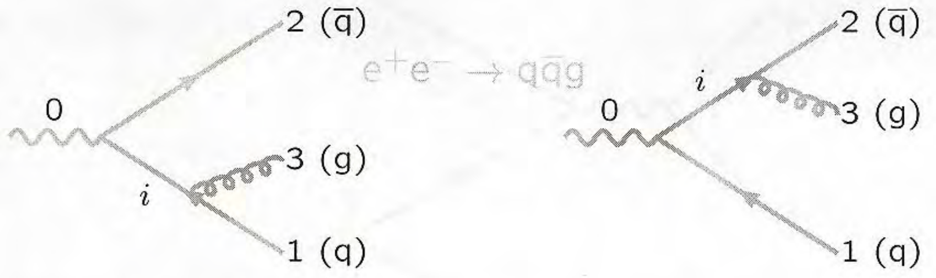
KLN cancellation theorem: $M(0) = M_0$

Two possible methods to get a finite answer exists: Phase space slicing (introducing arbitrary cutoff δ « 1) or subtraction. NLO will provide a more accurate integrated cross section

BUT neither approach uses only positive definite quantities
AND provides no obvious event generator implementation
AND gives no trivial connection to physical events.

(T. Sjöstrand)

From ME's to Parton Showers



$$x_j = 2E_j / E_{CM} \quad \Rightarrow \quad x_1 + x_2 + x_3 = 2$$

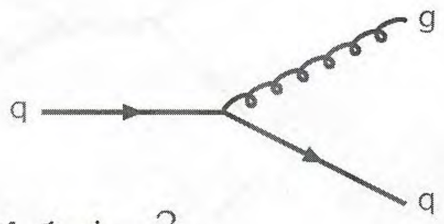
$$m_q = 0: \quad \frac{1}{\sigma_0} \frac{d\sigma_{ME}}{dx_1\, dx_2} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

rewrite for $x_2 \to 1$: (i.e. in the q-g collinear limit)

$$1 - x_2 = \frac{m_{13}^2}{E_{cm}^2} = \frac{Q^2}{E_{cm}^2}$$

$$x_1 \approx z$$

$$x_3 \approx 1 - z$$



$$\Rightarrow \quad d\mathcal{P} = \frac{d\sigma}{\sigma_0} \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1 + z^2}{1 - z} dz$$

generalizes to

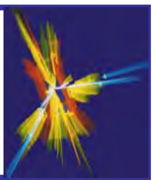
$$d\mathcal{P}_{a \to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \to bc}(z)\, dz$$

$$P_{q \to qg} = \frac{4}{3} \frac{1 + z^2}{1 - z}$$

$$P_{g \to gg} = 3 \frac{(1 - z(1 - z))^2}{z(1 - z)}$$

$$P_{g \to q\bar{q}} = \frac{n_f}{2} (z^2 + (1 - z)^2)$$

n_f = number of quark flavours



The Sudakov Form Factor

(T. Sjöstrand)

Conservation of total probability:

$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$

"multiplicativeness" in "time" evolution:

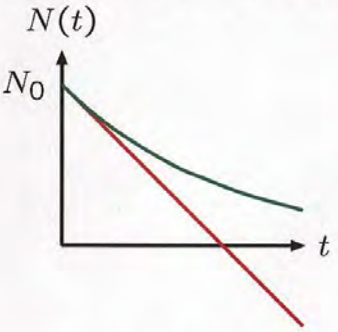
$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$

Subdivide further, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})\right) \\
&= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})\right) \\
&= \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
\end{aligned}$$

$$\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\, \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

Example: radioactive decay of nucleus



naively: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN_0 \Rightarrow N(t) = N_0\,(1 - ct)$

depletion: a given nucleus can only decay once

correctly: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN(t) \Rightarrow N(t) = N_0\,\exp(-ct)$

generalizes to: $N(t) = N_0\,\exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

or: $\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -c(t)\,N_0\,\exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

sequence allowed: $\text{nucleus}_1 \to \text{nucleus}_2 \to \text{nucleus}_3 \to \ldots$

Correspondingly, with $Q \sim 1/t$ (Heisenberg)

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z\, \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_{\max}} \frac{\mathrm{d}Q'^2}{Q'^2}\int \frac{\alpha_s}{2\pi} P_{a\to bc}(z')\,\mathrm{d}z'\right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c}\int \mathrm{d}Q^2 \int dz\, \mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo*

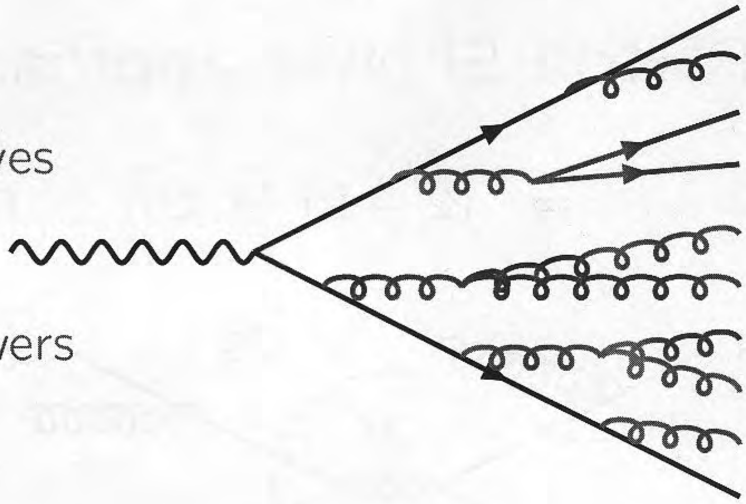
Parton shower

(T. Sjöstrand)

Iteration gives

final-state

parton showers

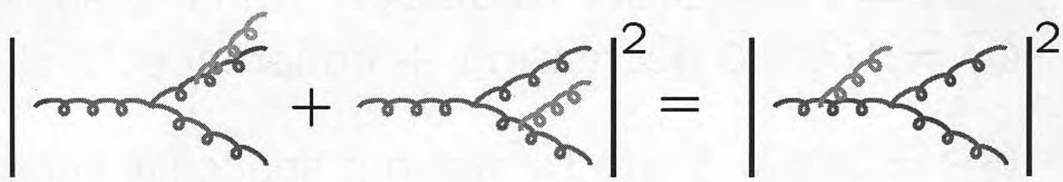


Sudakov form factor

$$\mathcal{P}^{\mathrm{corr}}(Q^2) = \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}Q^2} \exp\left(- \int_{Q^2}^{Q^2_{\mathrm{max}}} \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}Q^2} \mathrm{d}Q^2 \right)$$

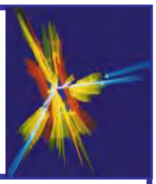
(cf. radioactive decay; 'time' ordering);
compensated by subsequent branchings

Coherence $\Rightarrow$ angular ordering



Emission scale: $\alpha_s(p_\perp^2) = \alpha_s(z(1-z)m^2)$

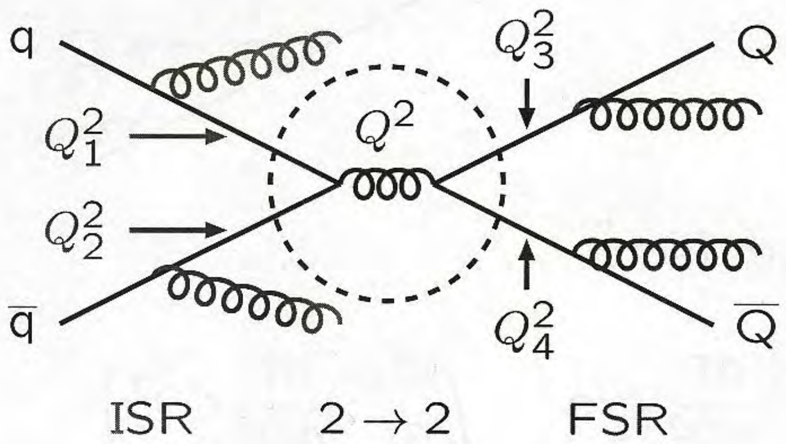
Shower evolution cut-off at some lower scale from which the hadronisation takes over. The soft and collinear cut-off mass $m_0 = \min(m_{ij}) \approx 1$ GeV usually.



Parton Shower approach

(T. Sjöstrand)

$2 \to n \;=\; (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$



$2 \to 2$ = hard scattering (on-shell)

$$\sigma = \iiint \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}\hat{t}\, f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

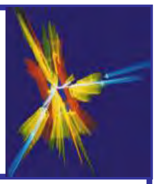
FSR = Final-State Radiation; timelike shower
$Q_i^2 = M^2 > 0$ decreasing + coherence

ISR = Initial-State Radiation; spacelike shower
$Q_i^2 = -M^2 > 0$ increasing + ~ coherence
backwards evolution: start at hard scattering

Do not doublecount!! $Q^2 > Q_1^2, Q_2^2, Q_3^2, Q_4^2$.

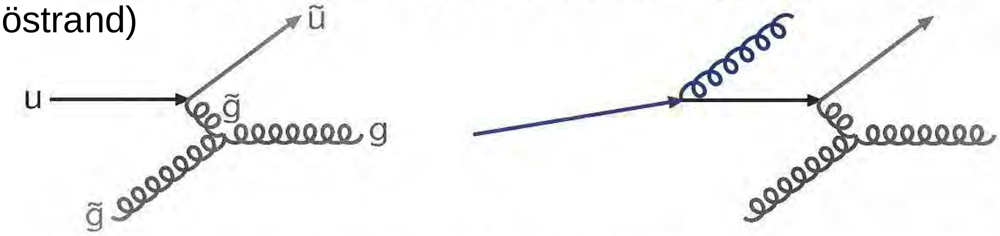
The $2 \to 2$ scattering = short distance interaction i.e. high Q^2.

Backwards evolution



Backwards evolution is viable and ∼equivalent alternative:
start at hard interaction and trace what happened "before"

(T. Sjöstrand)



Monte Carlo approach, based on *conditional probability*: recast

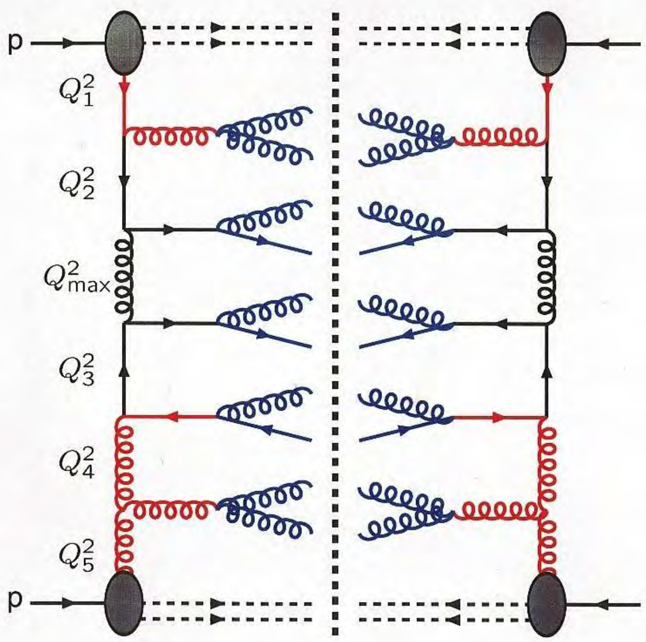
$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2)\,\frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

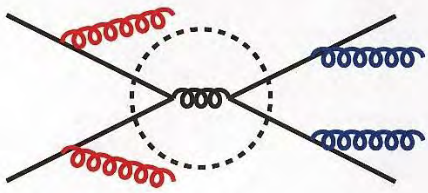
with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z\, \frac{x' f_a(x',t)}{x f_b(x,t)}\,\frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

then solve for *de*creasing t, i.e. backwards in time,
starting at high Q^2 and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Ladder representation combines whole event: cf. previously:





One possible
Monte Carlo order:

1) Hard scattering

2) Initial-state shower
from center outwards

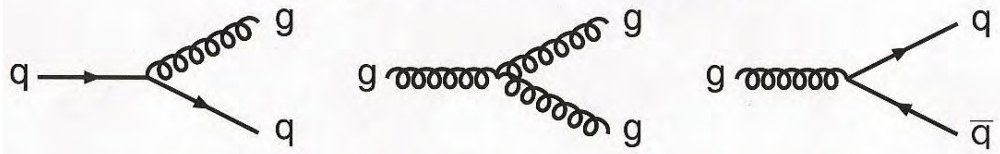
3) Final-state showers

DGLAP: $Q_{max}^2 > Q_1^2 > Q_2^2 \sim Q_0^2$
$Q_{max}^2 > Q_3^2 > Q_4^2 > Q_5^2 \sim Q_0^2$

Shower algorithms

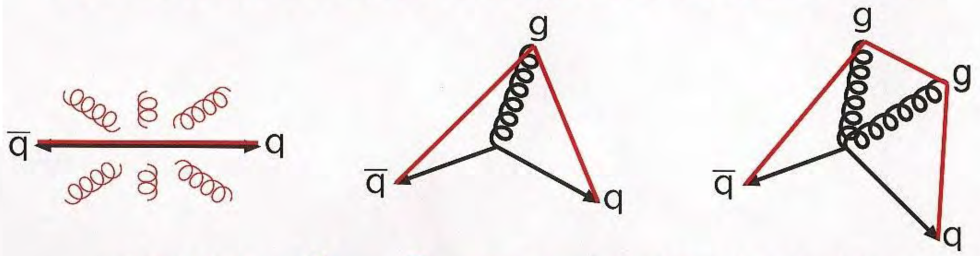
Three main approaches to showering in common use:
(T. Sjöstrand)

Two are based on the standard shower language of $a \to bc$ successive branchings:



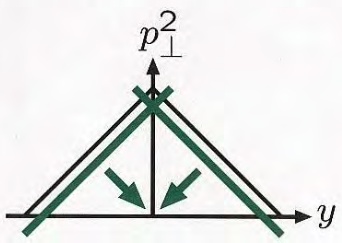
HERWIG: $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$
PYTHIA: $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

One is based on a picture of dipole emission $ab \to cde$:



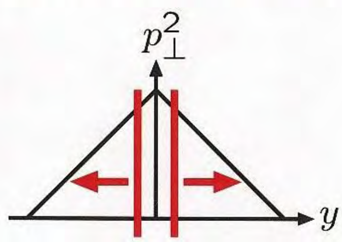
ARIADNE: $Q^2 = p_\perp^2$; FSR mainly, ISR is primitive;

PYTHIA: $Q^2 = m^2$



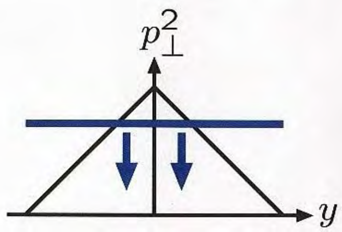
large mass first
⇒ "hardness" ordered
coherence brute force
covers phase space
ME merging simple
g → qq̄ simple
not Lorentz invariant
no stop/restart
ISR: $m^2 \to -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



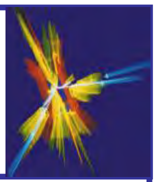
large angle first
⇒ hardness not ordered
coherence inherent
gaps in coverage
ME merging messy
g → qq̄ simple
not Lorentz invariant
no stop/restart
ISR: $\theta \to \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first
⇒ "hardness" ordered
coherence inherent

covers phase space
ME merging simple
g → qq̄ messy
Lorentz invariant
can stop/restart
ISR: more messy



(T. Sjöstrand)

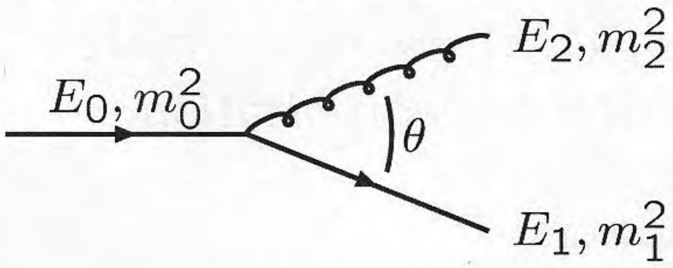
Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z \cdot \text{(Sudakov)}$$

but

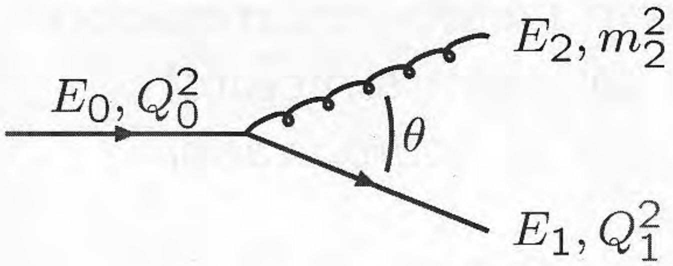
Final-state showers: Q^2 timelike ($\approx m^2$)



decreasing E
decreasing m^2
decreasing θ

daughters on equal footing, both $m^2 \geq 0$
$Q^2, z, \ldots$ choice gives several algorithms

Initial-state showers: Q^2 spacelike ($\approx -m^2$)



decreasing E
increasing Q^2
increasing θ

daughters unequal, one $m^2 \geq 0$, one $m^2 < 0$

$\Rightarrow$ kinematics & coherence more complicated

(T. Sjöstrand)

ME vs. PS

ME : Matrix Elements

- \+ systematic expansion in α_s (‘*exact*’)
- \+ powerful for multiparton Born level
- \+ flexible phase space cuts
- – loop calculations very tough
- – negative cross section in collinear regions
  $\Rightarrow$ unpredictive jet/event structure
- – *no easy match to hadronization*

PS : Parton Showers

- – approximate, to LL (or NLL)
- – main topology not predetermined
  $\Rightarrow$ inefficient for exclusive states
- \+ process-generic $\Rightarrow$ simple multiparton
- \+ Sudakov form factors/resummation
  $\Rightarrow$ sensible jet/event structure
- \+ *easy to match to hadronization*

Marriage desirable! But how?

Problems:
- gaps in coverage?
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

(T. Sjöstrand)

Matrix Element & Parton Shower merging

= smooth transition ME/PS, no sharp edge.
+ emissions can cover full phase space
− coherence not straightforward

Want to reproduce

$$W^{\mathrm{ME}} = \frac{1}{\sigma(\mathrm{LO})} \frac{\mathrm{d}\sigma(\mathrm{LO}+\mathrm{g})}{\mathrm{d(phasespace)}}$$

by shower generation + correction procedure

$$\overbrace{W^{\mathrm{ME}}}^{\text{wanted}} = \overbrace{W^{\mathrm{PS}}}^{\text{generated}} \overbrace{\frac{W^{\mathrm{ME}}}{W^{\mathrm{PS}}}}^{\text{correction}}$$

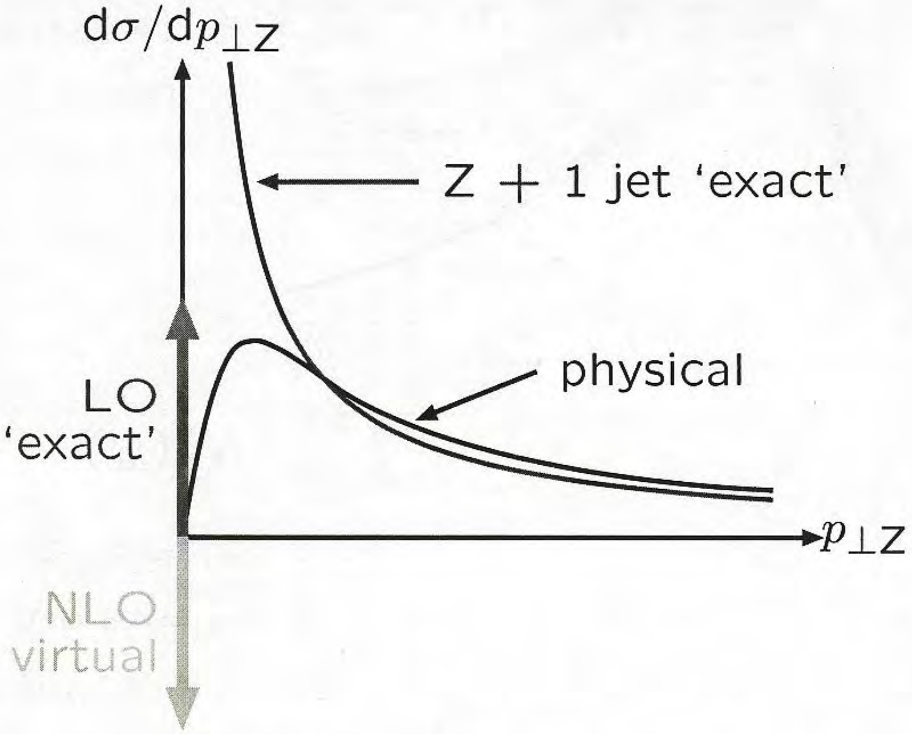
Comments:

- Do not normalize W^{ME} to $\sigma(\mathrm{NLO})$, since extra work without clear gain (expect radiation also in events added by K-factor ≥ 1)
- Exponentiate ME correction by shower Sudakov form factor:

$$W^{\mathrm{PS}}_{\mathrm{actual}}(Q^2) = W^{\mathrm{ME}}(Q^2) \exp\left(- \int_{Q^2}^{Q^2_{\mathrm{max}}} W^{\mathrm{ME}}(Q'^2)\, \mathrm{d}Q'^2 \right)$$

(T. Sjöstrand)

Initial-state showers merging

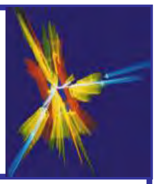


resummation: physical $p_{\perp Z}$ spectrum
shower: ditto + accompanying jets (exclusive)

Merged with matrix elements for
$q\bar{q} \to (\gamma^*/Z^0/W^\pm)g$ and $qg \to (\gamma^*/Z^0/W^\pm)q'$:
(G. Miu & TS, PLB449 (1999) 313)

$$\left(\frac{W^{ME}}{W^{PS}}\right)_{q\bar{q}'\to gW} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4} \leq 1$$

$$\left(\frac{W^{ME}}{W^{PS}}\right)_{qg\to q'W} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4} < 3$$

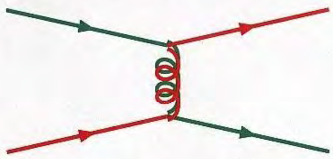


What is multiple interactions?

(T. Sjöstrand)

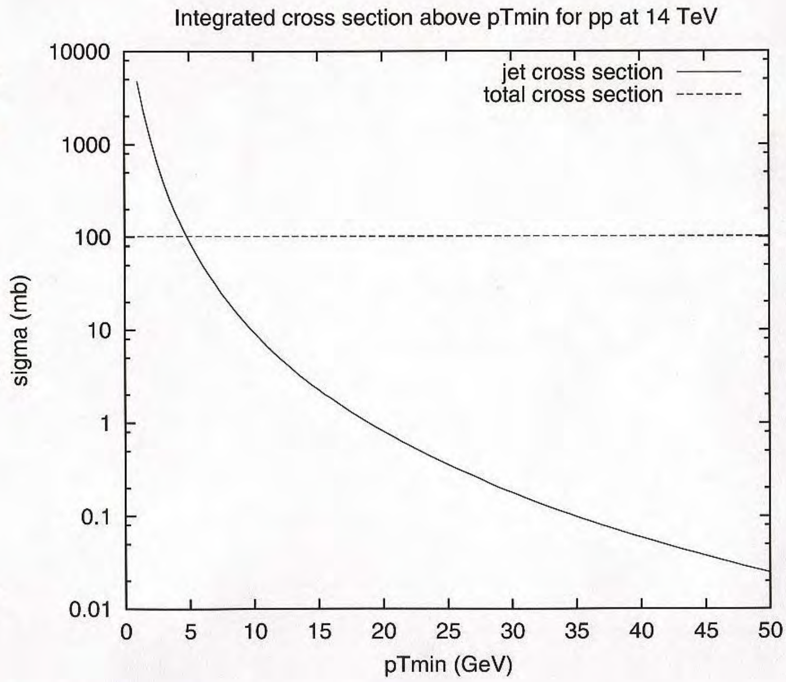
Cross section for $2 \to 2$ interactions is dominated by t-channel gluon exchange, so diverges like $\mathrm{d}\sigma/\mathrm{d}p_\perp^2 \approx 1/p_\perp^4$ for $p_\perp \to 0$.

integrate QCD $2 \to 2$



$qq' \to qq'$
$q\bar{q} \to q'\bar{q}'$
$q\bar{q} \to gg$
$qg \to qg$
$gg \to gg$
$gg \to q\bar{q}$

with CTEQ 5L PDF's



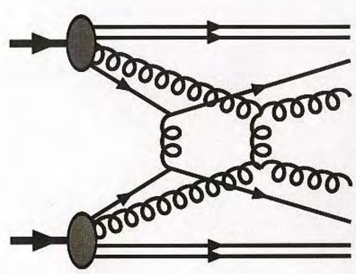
So $\sigma_{\mathrm{int}}(p_{\perp\mathrm{min}}) > \sigma_{\mathrm{tot}}$ for $p_{\perp\mathrm{min}} \lesssim 5$ GeV

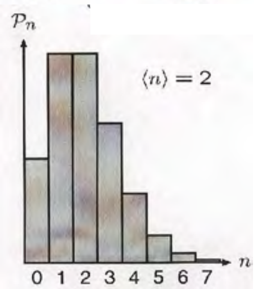
Half a solution: many interactions per event

$$\sigma_{\mathrm{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\mathrm{int}} = \sum_{n=0}^{\infty} n\,\sigma_n$$

$$\sigma_{\mathrm{int}} > \sigma_{\mathrm{tot}} \iff \langle n \rangle > 1$$





If interactions occur independently then **Poissonian statistics**

$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

but energy–momentum conservation
$\Rightarrow$ large n suppressed

Multiple interactions

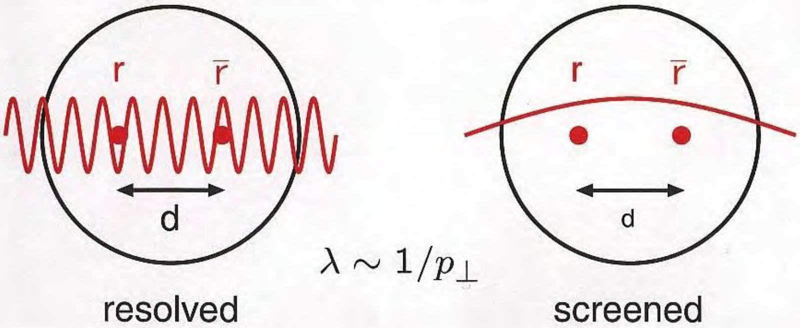
(T. Sjöstrand)

Other half of solution:
perturbative QCD not valid at small $p_\perp$ since q, g not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp\mathrm{min}} \simeq \frac{\hbar}{r_\mathrm{p}} \approx \frac{0.2\ \mathrm{GeV}\cdot\mathrm{fm}}{0.7\ \mathrm{fm}} \approx 0.3\ \mathrm{GeV} \simeq \Lambda_\mathrm{QCD}$$

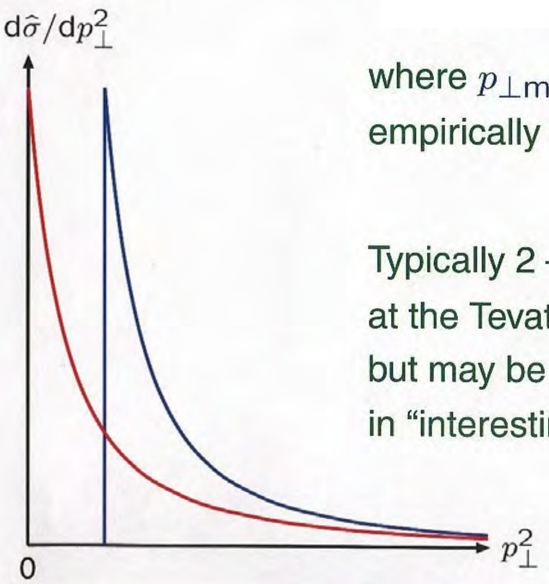
. . . but better replace r_p by (unknown) colour screening length d in hadron



so modify

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} \propto \frac{\alpha_\mathrm{s}^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_\mathrm{s}^2(p_\perp^2)}{p_\perp^4}\,\theta\left(p_\perp - p_{\perp\mathrm{min}}\right) \quad \text{(simpler)}$$

$$\text{or} \to \frac{\alpha_\mathrm{s}^2(p_{\perp 0}^2 + p_\perp^2)}{(p_{\perp 0}^2 + p_\perp^2)^2} \quad \text{(more physical)}$$



where $p_{\perp\mathrm{min}}$ or $p_{\perp 0}$ are free parameters, empirically of order **2 GeV**

Typically 2 – 3 interactions/event at the Tevatron, 4 – 5 at the LHC, but may be more in “interesting” high-$p_\perp$ ones.

Modelling multiple interactions

(T. Sjöstrand)

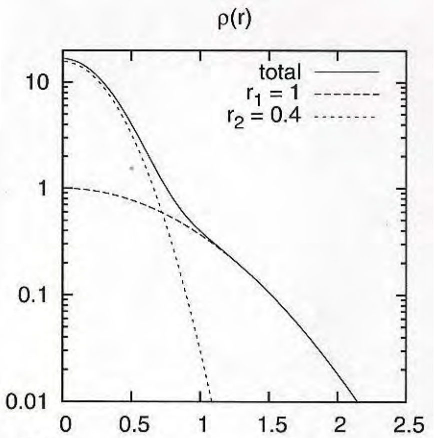
T. Sjöstrand, M. van Zijl, PRD36 (1987) 2019: first model(s) for event properties based on perturbative multiple interactions

(1) Simple scenario:

- Sharp cut-off at $p_{\perp\mathrm{min}}$ main free parameter
- Is only a model for nondiffractive events, i.e. for $\sigma_{\mathrm{nd}} \simeq (2/3)\sigma_{\mathrm{tot}}$
- Average number of interactions is $\langle n \rangle = \sigma_{\mathrm{int}}(p_{\perp\mathrm{min}})/\sigma_{\mathrm{nd}}$
- Interactions occur almost independently, i.e.
  Poissonian statistics $\mathcal{P}_n = \langle n \rangle^n e^{-\langle n \rangle}/n!$
  with fraction $\mathcal{P}_0 = e^{-\langle n \rangle}$ pure low-$p_\perp$ events
- Interactions generated in ordered sequence $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \ldots$
  by "Sudakov" trick (what happens "first"?)

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_{\perp i}} = \frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \exp\left[-\int_{p_\perp}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p'_\perp}\mathrm{d}p'_\perp\right]$$

- Momentum conservation in PDF's $\Rightarrow$ $\mathcal{P}_n$ narrower than Poissonian
- Simplify after first interaction: only gg or q$\bar{\mathrm{q}}$ outgoing, no showers, ...



(2) More sophisticated scenario:

- Smooth turn-off at $p_{\perp 0}$ scale
- Require ≥ 1 interaction in an event
- Hadrons are extended,
  e.g. double Gaussian ("hot spots"):

$$\rho_{\mathrm{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

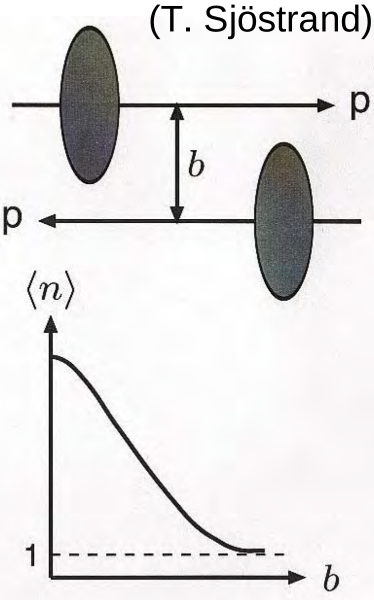
where $r_2 \neq r_1$ represents "hot spots"

Multiple interactions

(T. Sjöstrand)

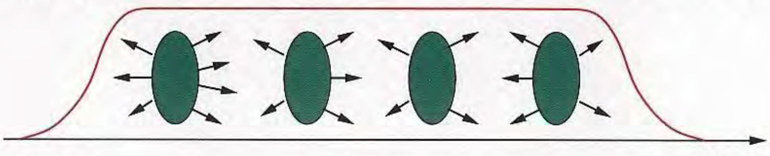
- Events are distributed in impact parameter b
- Overlap of hadrons during collision

$$\mathcal{O}(b) = \int \mathrm{d}^3\mathbf{x}\, \mathrm{d}t\, \rho_{1,\mathrm{matter}}^{\mathrm{boosted}}(\mathbf{x},t)\rho_{2,\mathrm{matter}}^{\mathrm{boosted}}(\mathbf{x},t)$$



- Average activity at b proportional to $\mathcal{O}(b)$
  $\Rightarrow$ central collisions normally more active
  $\Rightarrow$ $\mathcal{P}_n$ broader than Poissonian
- More time-consuming $(b, p_\perp)$ generation
- Need for simplifications remains

(3) HERWIG
Soft Underlying Event (SUE), based on UA5 Monte Carlo



- Distribute a ($\sim$ negative binomial) number of clusters independently in rapidity and transverse momentum according to parametrization/extrapolation of data
- modify for overall energy/momentum/flavour conservation
- no minijets; correlations only by cluster decays

(4) Jimmy (HERWIG add-on)
- similar to PYTHIA (2) above; but details different
- matter profile by electromagnetic form factor
- no $p_\perp$-ordering of emissions, no rescaling of PDF: abrupt stop when (if) run out of energy

(5) Phojet/DTUjet
- comes from "historical" tradition of soft physics of "cut Pomerons" $\approx p_\perp \to 0$ limit of multiple interactions
- extended also to "hard" interactions similarly to PYTHIA

(T. Sjöstrand)

Hadronization/Fragmentation models

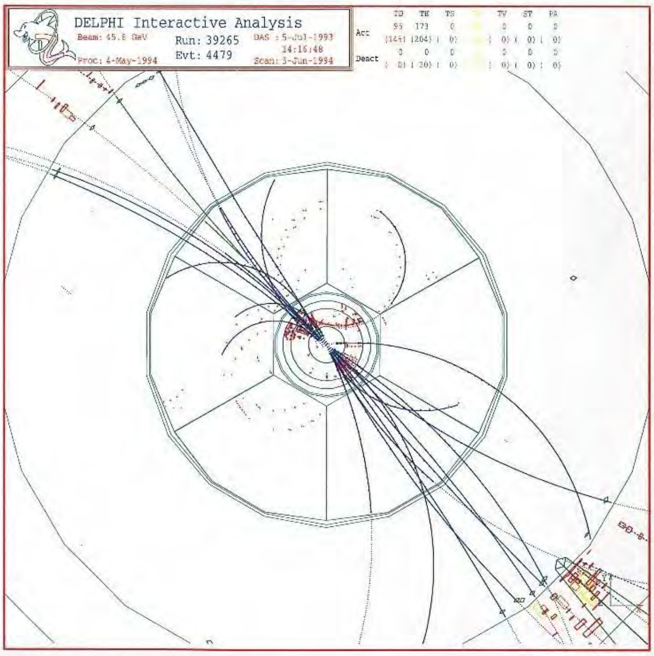
Perturbative → nonperturbative ⟹ not calculable from first principles!

Model building = ideology + “cookbook”

Common approaches:

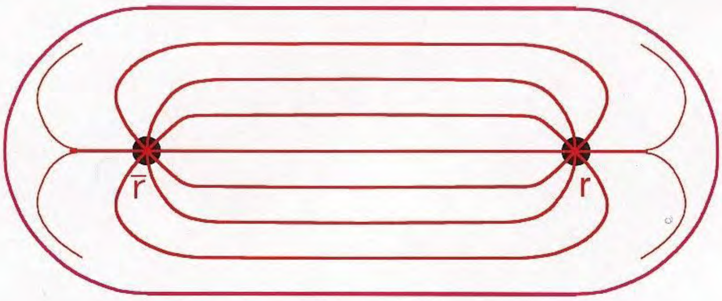
1) **String** Fragmentation
   (most ideological)
2) **Cluster** Fragmentation
   (simplest?)
3) **Independent** Fragmentation
   (most cookbook)
4) Local Parton–Hadron Duality
   (limited applicability)

Best studied in
$e^+e^- \to \gamma^*/Z^0 \to q\overline{q}$



The Lund String Model

In QCD, for large charge separation, field lines seem to be compressed to tubelike region(s) ⇒ **string(s)**



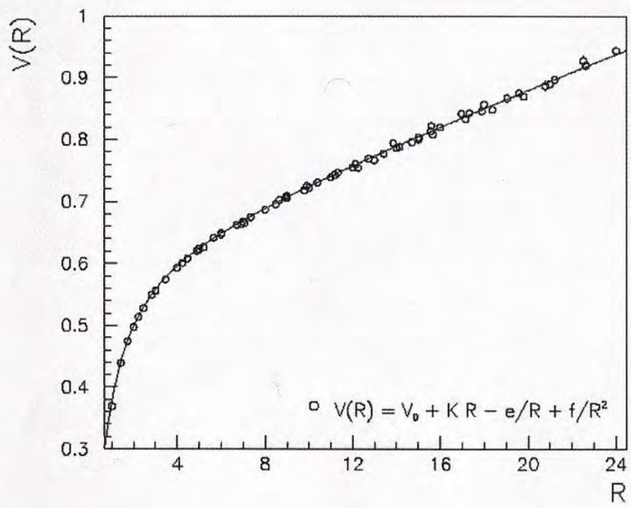
by self-interactions among soft gluons in the “vacuum”.

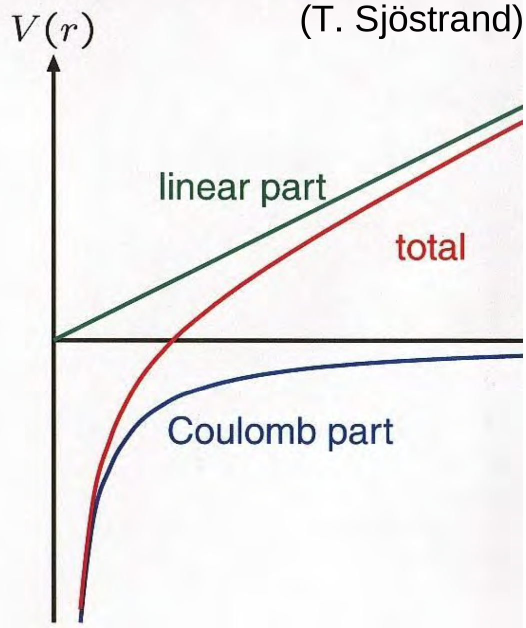
Gives linear confinement with string tension:

$$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \iff V(r) \approx \kappa r$$

Linear confinement confirmed e.g. by quenched lattice QCD

(T. Sjöstrand)





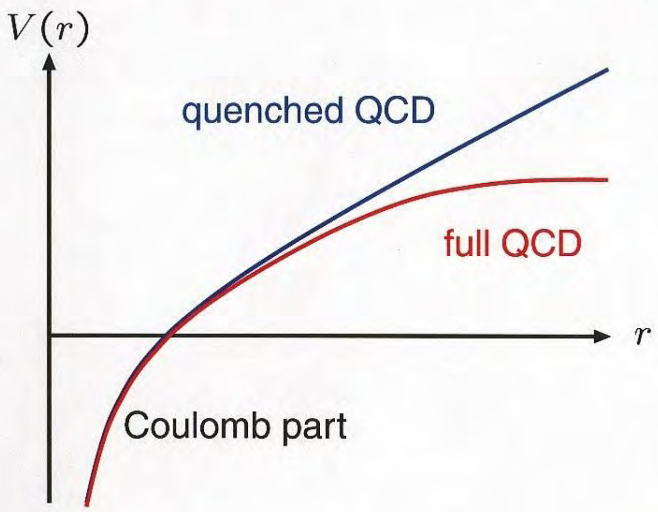
$$V(r) \approx -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r \approx -\frac{0.13}{r} + r$$

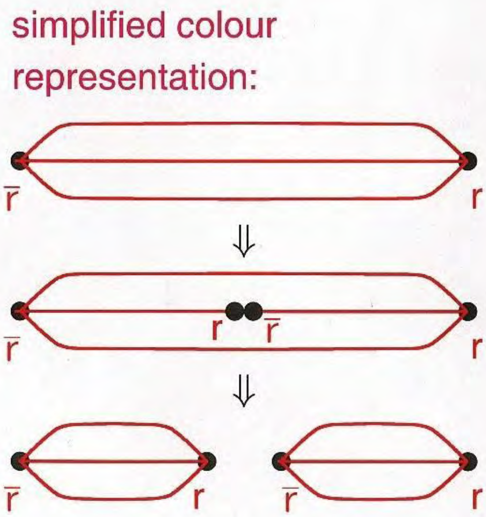
(for $\alpha_s \approx 0.5$, r in fm and V in GeV)

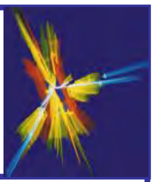
$V(0.4\,\mathrm{fm}) \approx 0$: Coulomb important for internal structure of hadrons,

Real world (??, or at least unquenched lattice QCD)

$\Longrightarrow$ nonperturbative string breakings gg ... $\to$ $q\overline{q}$

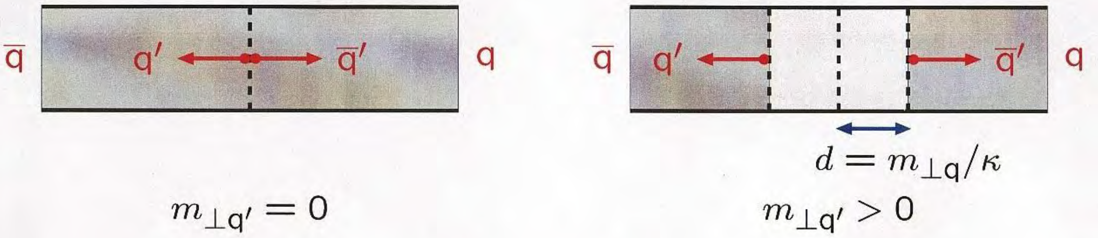






(T. Sjöstrand)

How does the string break?



String breaking modelled by tunneling:

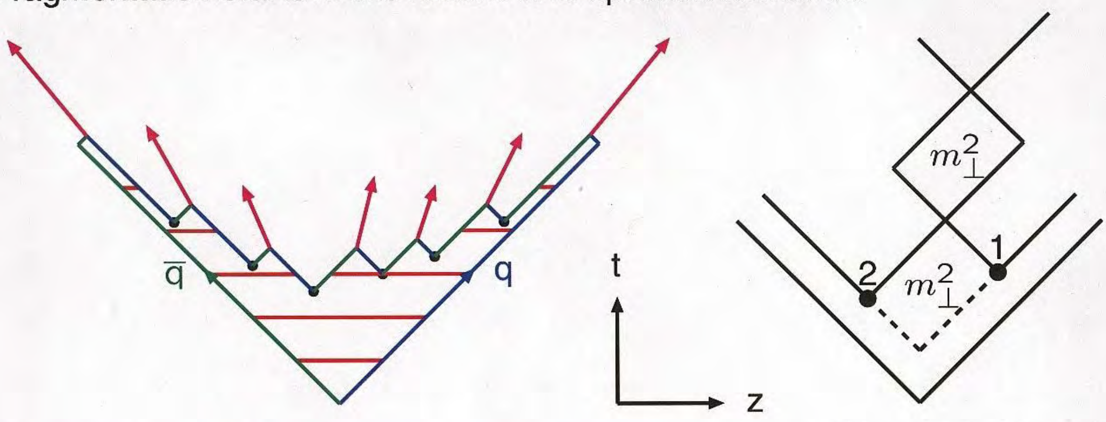
$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) = \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

1) common Gaussian $p_\perp$ spectrum

2) suppression of heavy quarks $u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$

3) diquark $\sim$ antiquark $\Rightarrow$ simple model for baryon production

Fragmentation starts in the middle and spreads outwards:



but breakup vertices causally disconnected

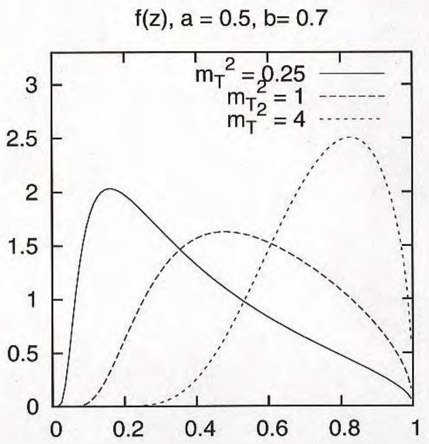
$\Rightarrow$ can proceed in arbitrary order

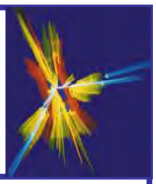
$\Rightarrow$ *left–right symmetry*

$$\begin{aligned} \mathcal{P}(1,2) &= \mathcal{P}(1) \times \mathcal{P}(1 \to 2) \\ &= \mathcal{P}(2) \times \mathcal{P}(2 \to 1) \end{aligned}$$

$\Rightarrow$ Lund symmetric fragmentation function

$f(z) \propto (1-z)^a \exp(-bm_\perp^2/z)/z$

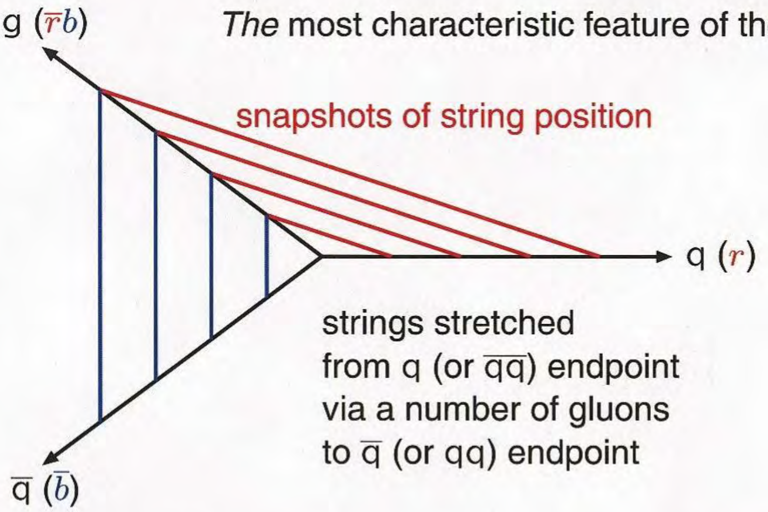




The Lund gluon picture

(T. Sjöstrand)

The most characteristic feature of the Lund model



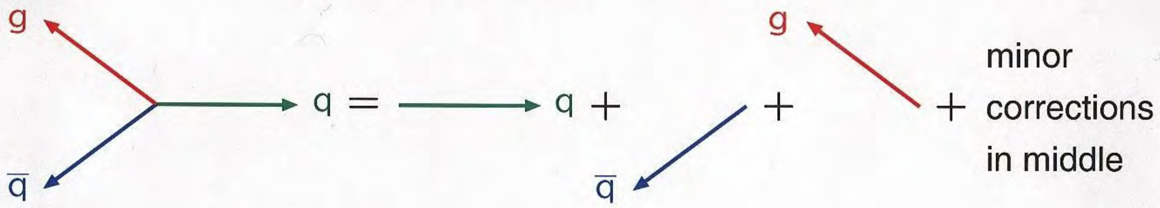
Gluon = kink on string, carrying energy and momentum

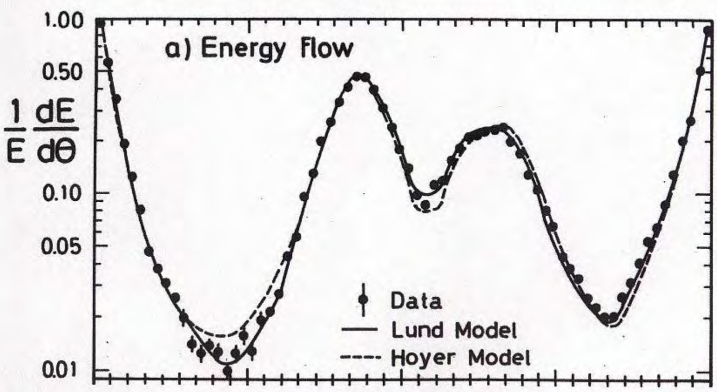
Force ratio gluon/ quark = 2, cf. QCD $N_C/C_F = 9/4, \to 2$ for $N_C \to \infty$

No new parameters introduced for gluon jets!

Independent fragmentation

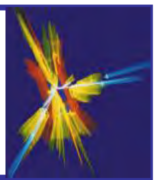
Based on a similar iterative ansatz as string, but





String effect
(JADE, 1980)
≈ coherence in
nonperturbative
context

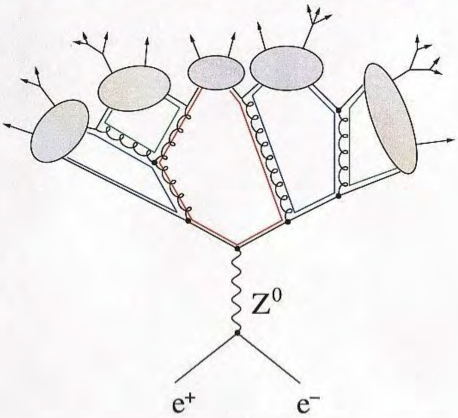
Further numerous and detailed tests at LEP favour string picture . . .

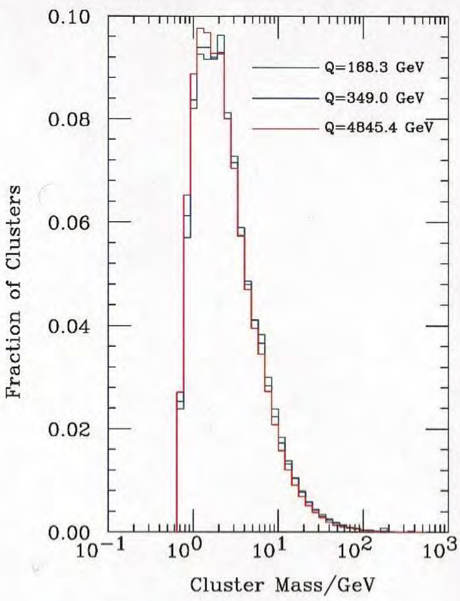


The HERWIG Cluster Model

(T. Sjöstrand)

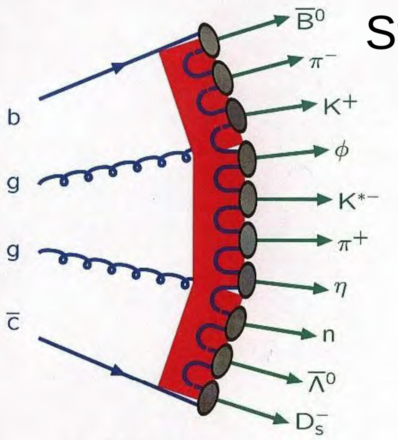
"Preconfinement":
colour flow is local
in coherent shower evolution

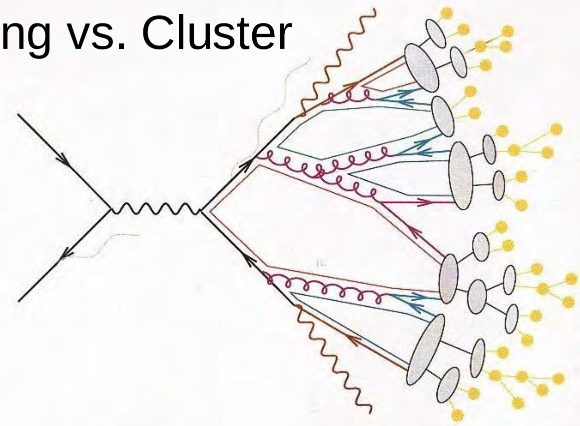




1) Introduce forced g $\rightarrow$ q$\overline{\text{q}}$ branchings
2) Form colour singlet clusters
3) Clusters decay isotropically to 2 hadrons according to phase space weight $\sim (2s_1+1)(2s_2+1)(2p^*/m)$

String vs. Cluster





program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful predictive	simple unpredictive
parameters	few	many
flavour composition	messy unpredictive	simple in-between
parameters	many	few

(T. Sjöstrand)

Local Parton–Hadron Duality

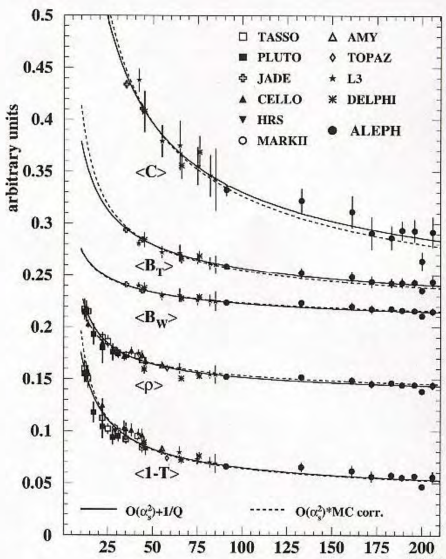
Analytic approach:
Run shower down to to $Q \approx \Lambda_{\mathrm{QCD}}$
(or m_{hadron}, if larger)

"Hard Line": each parton $\equiv$ one hadron

"Soft Line": local hadron density
$\propto$ parton density

describes momentum spectra $\mathrm{d}n/\mathrm{d}x_p$
and semi-inclusive particle flow,
but fails for identified particles

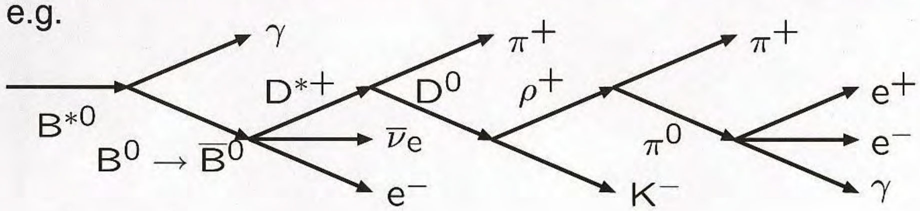
Not MC, **not** for arbitrary quantity



Decays

Unspectacular/ungrateful but necessary:
this is where most of the final-state particles are produced!
Involves hundreds of particle kinds and thousands of decay modes.

e.g.



- $\mathrm{B}^{*0} \to \mathrm{B}^0\gamma$: electromagnetic decay
- $\mathrm{B}^0 \to \overline{\mathrm{B}}^0$ mixing (weak)
- $\overline{\mathrm{B}}^0 \to \mathrm{D}^{*+}\overline{\nu}_{\mathrm{e}}\mathrm{e}^-$: weak decay, displaced vertex, $|\mathcal{M}|^2 \propto (p_{\overline{\mathrm{B}}}p_{\overline{\nu}})(p_{\mathrm{e}}p_{\mathrm{D}^*})$
- $\mathrm{D}^{*+} \to \mathrm{D}^0\pi^+$: strong decay
- $\mathrm{D}^0 \to \rho^+\mathrm{K}^-$: weak decay, displaced vertex, ρ mass smeared
- $\rho^+ \to \pi^+\pi^0$: ρ polarized, $|\mathcal{M}|^2 \propto \cos^2\theta$ in ρ rest frame
- $\pi^0 \to \mathrm{e}^+\mathrm{e}^-\gamma$: Dalitz decay, $m(\mathrm{e}^+\mathrm{e}^-)$ peaked

Dedicated programs, with special attention to polarization effects:
- EVTGEN: B decays
- TAUOLA: τ decays

(T. Sjöstrand)

Event generation structure

1) Initialization step

- select process(es) to study
- modify physics parameters: m_t, m_h, ...
- set kinematics constraints
- modify generator performance
- initialize generator
- book histograms

2) Generation loop

- generate one event at a time
- analyze it (or store for later use)
- add results to histograms
- print a few events

3) Finishing step

- print deduced cross-sections
- print/save histograms etc.

Final Words of Warning

[...] The Monte Carlo simulation has become the major means of visualization of not only detector performance but also of physics phenomena. So far so good. But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data.
[...] I am prepared to believe that the computer-literate generation (of which I am a little too old to be a member) is in principle no less competent and in fact benefits relative to us in the older generation by having these marvelous tools. They do allow one to look at, indeed visualize, the problems in new ways. But I also fear a kind of "terminal illness", perhaps traceable to the influence of television at an early age. There the way one learns is simply to passively stare into a screen and wait for the truth to be delivered. A number of physicists nowadays seem to do just this.

J.D. Bjorken